H-infinity Filtering for Sensor Networked Spacecraft Thermal Experiments

Chang Liu[a], Tao Liao, Dongliang Wu, Jihui Xie

Beijing Institute of Spacecraft Environment Engineering, Beijing, China

[a]Changliu.511@outlook.com

Keywords: H-infinity filtering; sensor network; spacecraft thermal experiments

Abstract: This paper studies the H-infinity filtering method for sensor networked spacecraft thermal experiments. By constructing Lyapunov function, sufficient conditions are derived for the desired filtering gains with the help of matrix transformations. In the end, an illustrative example of surplus temperature filtering during spacecraft thermal tests is presented for verifying the applicability of our proposed method.

1. Introduction

During the past years, senor networks have been attracting increasing attention for the theoretical significance and practical applications [1-2]. By applying the sensor networks, a group of sensors can work cooperatively with each other according to the specific network communication topologies, such that information exchanges can be achieved. Furthermore, the distributed filtering problem via sensor networks is becoming a hot research topic. Different from filtering methods with a single sensor, more robustness and reliability can be obtained by sensor networks [3-4].

On the other hand, the H-infinity theory has been a very powerful tool in control system analysis and synthesis procedures. In particular, when certain H-infinity can be satisfied, the disturbance attenuation can be obtained by desired H-infinity performance designs. To name a few, many remarkable results of H-infinity filtering problems have been reported in the literature and the references therein [5-6]. However, it should be pointed out that there are few results of the H-infinity filtering of spacecraft thermal experiments by sensor networks until now, which motivates us for this current study.

In this paper, we deal with the H-infinity filtering problem of sensor networked spacecraft thermal experiments. By model transformation, linear matrix inequality conditions are derived for the desired filtering gain design. In addition, the obtained theoretical results are further applied to the spacecraft ground thermal tests for demonstration.

The outline of the paper is presented as follows: the distributed H-infinity filtering problem is first introduced with essential preliminaries. Then, the main theoretical results of this paper are given accordingly. Furthermore, the illustrative simulations are provided for demonstrating the effectiveness of the design method and the concluding remarks are drawn accordingly.

The following notations are used in the paper:

$\mathbb{R}^n$ denotes the n dimensional Euclidean space.

$A-B<0$ means $A-B$ is negative definite.

$A\otimes B$ stands for the Kronecker product.

Diagf…g denotes a block-diagonal matrix.

The superscript "T" denotes the matrix transposition.

$*$ denotes the ellipsis for the terms that are introduced by symmetry.

All matrices are supposed to be compatible for algebraic operations.

2. Problem Formulation

Consider the target plant with the following dynamics:

  DOI: 10.25236/icemit.2019.002

$$\dot{x}(t) = Ax(t) + Bw(t),$$
$$z(t) = Lx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector; $w(t) \in \mathbb{R}^p$ denotes the external disturbance; $z(t) \in \mathbb{R}^m$ denotes the state vector to be estimated. A, B, L are all known constant matrices.

The sensor networks are applied, where the network topology is represented by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ with $\mathcal{V}$ and $\mathcal{E}$ being the sets of nodes and edges, respectively. As a result, the measured output by the sensor *i* can be obtained as follows:

$$y_i(t) = C_i x(t),$$
$$i = 1, 2, \ldots, N, \tag{2}$$

where $y_i(t)$ denotes the measured output; and C is a known constant matrix.

As a result, the distributed filters can be designed as follows:

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + \sum a_{ij} K_{ij} (C_j x(t) - C_j \hat{x}_j(t)),$$
$$\hat{z}_i(t) = L\hat{x}_i(t), \tag{3}$$

where $\hat{x}_i(t)$ denotes the *i*th sensor estimation, K_{ij} denotes the sensor estimator gains to be designed.

By letting

$$e_i(t) = x(t) - \hat{x}_i(t), \tag{4}$$

and

$$\bar{z}_i(t) = z(t) - \hat{z}_i(t), \tag{5}$$

it can be obtained that

$$\dot{e}_i(t) = Ae_i(t) - \sum a_{ij} K_{ij} C_j e_j(t) + Bw(t)$$
$$\bar{z}_i(t) = Le_i(t), \tag{6}$$

which can be rewritten as follows:

$$\dot{e}(t) = \bar{A}e(t) + \bar{B}w(t) - \bar{K}\bar{C}e(t),$$
$$\bar{z}(t) = \bar{\mathrm{L}}\, e(t), \tag{7}$$

where

$$
\begin{aligned}
e(t) &= [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T, \\
\bar{z}(t) &= [\bar{z}_1^T(t), \bar{z}_2^T(t), \ldots, \bar{z}_N^T(t)]^T, \\
\bar{A} &= (I_N \otimes A), \\
\bar{B} &= (B^T, B^T, \ldots, B^T)^T, \\
\bar{K} &= [a_{ij} K_{ij}]_{N \times N}, \\
\bar{C} &= \mathrm{diag}\{C_1, C_2, \ldots, C_N\}, \\
\bar{\mathrm{L}} &= (I_N \otimes L).
\end{aligned}
$$

As a result, it follows that

$$\dot{\xi}(t) = \mathcal{A}\xi(t) + \mathcal{B}w(t),$$
$$\bar{z}(t) = \mathcal{L}\xi(t), \tag{8}$$

where

$$\mathcal{A} = \begin{bmatrix} A & 0 \\ 0 & \bar{A} - \bar{K}\bar{C} \end{bmatrix},$$
$$\mathcal{B} = \begin{bmatrix} B \\ \bar{B} \end{bmatrix},$$
$$\mathcal{L} = \begin{bmatrix} 0 & \bar{L} \end{bmatrix},$$

and

$$\xi(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}.$$

Moreover, the H-infinity filtering is said to be achieved if it holds that

$$\|\bar{z}(t)\|^2 \le \gamma^2 \|w(\mathrm{t})\|^2 \tag{9}$$

which implies that γ is the H-infinity performance index parameter.

Before proceeding further, the following important Lemma is provided for later use.

Lemma 1: Given constant matrices S_1, S_2 and S_3, where $S_1^T = S_1$, and $S_2^T = S_2 > 0$, then

$$S_1 + S_3^T S_2 S_3 < 0$$

if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ * & -S_2 \end{bmatrix} < 0$$

or

$$\begin{bmatrix} -S_2 & S_3^T \\ * & -S_1 \end{bmatrix} < 0 .$$

3. Main Results

In this section, our proposed filter design procedure will be given with details.

Theorem 1: The H-infinity filtering can be achieved for system (1) with the given filter gains if there exists a matrix $P = \mathrm{diag}\{P_1, \mathcal{P}_2\} > 0$ with $\mathcal{P}_2 = \mathrm{diag}\{P_2, P_3, \ldots, P_{N+1}\}$ such that the following linear matrix inequality can be satisfied:

$$\begin{bmatrix} P_1 A + A^T P_1 & 0 & P_1 B \\ * & \begin{matrix} \mathcal{P}_2 \bar{A} + \bar{A}^T \mathcal{P}_2 - \mathcal{P}_2 \bar{K}\bar{C} \\ -\bar{C}^T \bar{K}^T \mathcal{P}_2 + \bar{L}^T \bar{L} \end{matrix} & \mathcal{P}_2 \bar{B} \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{10}$$

Proof: Choose the following Lyapunov function:

$$V(t) = \xi^T(t) P \xi(t) \tag{11}$$

Then, one can obtain that

$$
\begin{aligned}
\dot{V}(t) &= \dot{\xi}^T(t)P\xi(t)+\xi^T(t)P\dot{\xi}(t)\\
&=2\xi^T(t)P\mathcal{A}\xi(t)+2\xi^T(t)P\mathcal{B}w(t)\\
&=2[x^T(t),e^T(t)]\begin{bmatrix} P_1 & 0\\ 0 & \mathcal{P}\end{bmatrix}\times\\
&\begin{bmatrix} A & 0\\ 0 & \bar{A}-\bar{K}\bar{C}\end{bmatrix}\begin{bmatrix} x(t)\\ e(t)\end{bmatrix}\\
&+2[x^T(t),e^T(t)]\begin{bmatrix} P_1 & 0\\ 0 & \mathcal{P}\end{bmatrix}\times\\
&\begin{bmatrix} B\\ \bar{B}\end{bmatrix}w(t).
\end{aligned} \tag{12}
$$

Furthermore, the following index is given:

$$J=\|\bar{z}(t)\|^2-\gamma^2\|w(\mathrm{t})\|^2.$$

Then, by applying the H-infinity performance and lemma 1, the proof can be completed.

Remark 1: It is worth mentioning that the obtained sufficient conditions are in the form of strict LMIs, which can be easily solved by the mathematical tools, such as the MATLAB. When the LMIs have feasible solutions, the desired filtering gains can be obtained by the following Theorem.

Remark 2: The established conditions can be easily extended to the cases with time delays by certain time delay LMI conditions.

Theorem 2: The H-infinity filtering can be achieved for system (1) if there exist matrices $P=\mathrm{diag}\{P_1,\mathcal{P}_2\}>0$ with $\mathcal{P}_2=\mathrm{diag}\{P_2,P_3,\ldots,P_{N+1}\}$ and $\mathcal{K}$ such that the following linear matrix inequality can be satisfied:

$$
\begin{bmatrix}
P_1A+A^TP_1 & 0 & P_1B\\
* & \begin{matrix}\mathcal{P}_2\bar{A}+\bar{A}^T\mathcal{P}_2-\mathcal{K}\bar{C}\\ -\bar{C}^T\mathcal{K}^T+\bar{L}^T\bar{L}\end{matrix} & \mathcal{P}_2\bar{B}\\
* & * & -\gamma^2I
\end{bmatrix}<0 \tag{13}
$$

In addition, the distributed filter gains can be obtained by the following:

$$\bar{K}=\mathcal{P}_2^{-1}\mathcal{K} \tag{14}$$

which implies that

$$\bar{K}=[a_{ij}K_{ij}]_{N\times N},$$

and the corresponding K_{ij} can be calculated by the matrix transformations.

Proof: Let $\bar{K}=\mathcal{P}_2^{-1}\mathcal{K}$ and the results can be directly obtained from Theorem 1.

4. Numerical Example

In this section, a simulation example is given with application to spacecraft thermal tests for showing the effectiveness of our proposed approach.

Consider the following target surplus temperature dynamics between the target equipment and the whole spacecraft during the ground thermal tests:

$$\frac{d\theta(t)}{dt}+\frac{K}{C}\theta(t)=\frac{1}{C}Q, \tag{15}$$

where $\theta(t)$ denotes the surplus temperature; K and C denote the heat conductivity and the heat capacity between the target equipment and the spacecraft, respectively. Q denotes the total power of the heat source.

By model transformation with the equilibrium of $\theta(t)$ as θ^*, system (15) can be rewritten in the form of system (1), where

$$A = -\frac{K}{C}. \tag{16}$$

As a result, in the conducted simulation, the model parameters of the above surplus temperature model are selected as: *K=5* and *C=90.* The H-infinity performance parameter is given as γ=5.

Moreover, the adjacency structure of the sensor networks that denoting the communication network topology is set as follows:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Then, the sensor parameters are selected as

$$C_1 = 0.25,$$
$$C_2 = 0.18,$$
$$C_3 = 0.2,$$
$$C_4 = 0.3,$$

and

$$D_1 = 0.2,$$
$$D_2 = 0.2,$$
$$D_3 = 0.2,$$
$$D_4 = 0.2,$$

As a result, the desired distributed filter gains K_{ij} can be calculated by Theorem 2 as follows:

$$K_{11} = -0.0794,$$
$$K_{21} = 0.0448,$$
$$K_{22} = 2.6287,$$
$$K_{31} = 0.0448,$$
$$K_{32} = -0.1893,$$
$$K_{33} = 2.3691,$$
$$K_{43} = -0.1707,$$
$$K_{44} = 1.5787.$$

From the simulation results, one can observe that our designed filter gains can ensure the given H-infinity performance, which supports our design method.

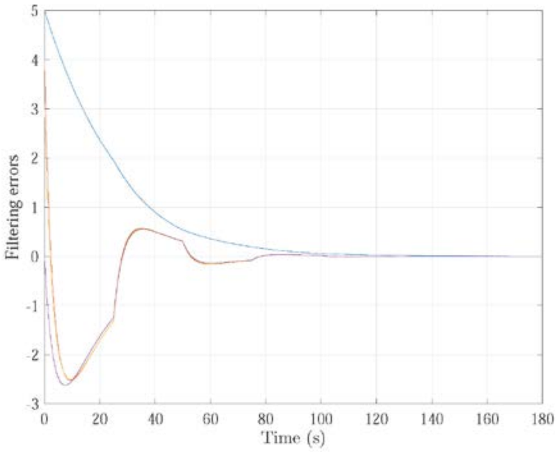

Fig. 1. Distributed filtering errors of the sensor networks.

5. Conclusions

In this paper, we deal with the distributed filtering problems of sensor networked spacecraft thermal experiments by employing the H-infinity performance. By constructing the Lyapunov function, sufficient criteria can be established such that the resulting filtering error system can achieve the desired performance. Finally, the simulations with spacecraft thermal tests is given for showing the merit of the proposed filtering method.

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